Exercise 7D

1 $\theta = t^2 - 3t$

$\frac{d\theta}{dt} = 2t - 3$

2 $A = 2\pi r$

$\frac{dA}{dr} = 2\pi$

3 $r = \frac{12}{t} = 12t^{-1}$

$\frac{dr}{dt} = -12t^{-2} = -\frac{12}{t^2}$

When $t = 3$,

$\frac{dr}{dt} = -\frac{12}{3^2} = -\frac{12}{9} = -\frac{4}{3}$

4 $A = 4\pi r^2$

$\frac{dA}{dr} = 8\pi r$

When $r = 6$,

$\frac{dA}{dr} = 8\pi \times 6$

$= 48\pi$

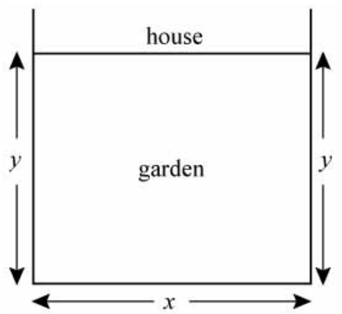
5 $s = t^2 + 8t$

$\frac{ds}{dt} = 2t + 8$

When $t = 5$,

$\frac{ds}{dt} = 2 \times 5 + 8 = 18$

6 a



Let the width of the garden be x m.

Then $x + 2y = 80$

$x = 80 - 2y$ (1)

Area $A = xy$

$= y(80 - 2y)$

$= 80y - 2y^2$

6 b $\frac{dA}{dy} = 80 - 4y$

Putting $\frac{dA}{dy} = 0$ for maximum area:

$80 - 4y = 0$

$y = 20$

Substituting in (1): $x = 40$

So area = 40 m × 20 m = 800 m^2

7 a Total surface area $= 2\pi rh + 2\pi r^2$

$2\pi rh + 2\pi r^2 = 600\pi$

$rh = 300 - r^2$

Volume $= \pi r^2 h = \pi r(rh) = \pi r(300 - r^2)$

So $V = 300\pi r - \pi r^3$

b For maximum volume, $\frac{dV}{dr} = 0$

$\frac{dV}{dr} = 300\pi - 3\pi r^2$

$300\pi - 3\pi r^2 = 0$

$r^2 = 100$

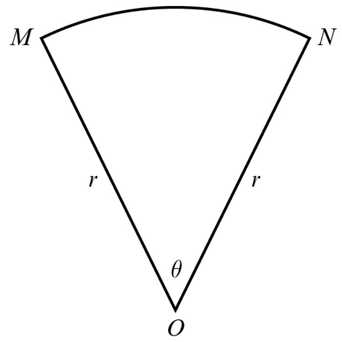
$r = 10$

Substituting $r = 10$ into V gives:

$V = 300\pi \times 10 - \pi \times 10^3 = 2000\pi$

Maximum volume $= 2000\pi$ cm^3

8 a



Let angle $MON = \theta$ radians

Then perimeter $P = 2r + r\theta$ (**1**)

and area $A = \frac{1}{2}r^2\theta$

Area = 100 cm^2

$\frac{1}{2}r^2\theta = 100$

$r\theta = \frac{200}{r}$

Substituting into (**1**) gives:

$P = 2r + \frac{200}{r}$ (**2**)

Since area of circle > area of sector

$\pi r^2 > 100$

$r > \sqrt{\frac{100}{\pi}}$

b For minimum perimeter, $\frac{dP}{dr} = 0$

$\frac{dP}{dr} = 2 - \frac{200}{r^2}$

$2 - \frac{200}{r^2} = 0$

$r^2 = 100$

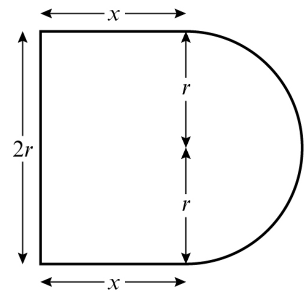
$r = 10$

Substituting into (2) gives:

$P = 20 + \frac{200}{10} = 40$

Minimum perimeter = 40 cm

9 a



Let the rectangle have dimensions $2r$ by x cm.

Perimeter of figure = $(2r + 2x + \pi r)$ cm

Perimeter = 40 cm, so

$2r + 2x + \pi r = 40$

$x = \frac{40 - \pi r - 2r}{2}$

Area = rectangle + semicircle

$= 2rx + \frac{1}{2}\pi r^2$

Substituting $x = \frac{40 - \pi r - 2r}{2}$:

$A = r(40 - \pi r - 2r) + \frac{1}{2}\pi r^2$

$= 40r - 2r^2 - \frac{1}{2}\pi r^2$

b For maximum area, $\frac{dA}{dr} = 0$:

$\frac{dA}{dr} = 40 - 4r - \pi r$

$40 - 4r - \pi r = 0$

$r = \frac{40}{4 + \pi}$

When $r = \frac{40}{4 + \pi}$,

$A = 40 \times \frac{40}{4+\pi} - 2\left(\frac{40}{4+\pi}\right)^2 - \frac{1}{2}\pi\left(\frac{40}{4+\pi}\right)^2$

$= \frac{1600}{4+\pi} - \left(2 + \frac{1}{2}\pi\right)\left(\frac{40}{4+\pi}\right)^2$

$= \frac{1600}{4+\pi} - \frac{4+\pi}{2} \times \frac{1600}{(4+\pi)^2}$

$= \frac{1600}{4+\pi} - \frac{800}{4+\pi}$

$= \frac{800}{4+\pi}$

So maximum area = $\frac{800}{4+\pi}$ cm^2

10 a Total length of wire = $(18x + 14y)$ mm
Length = 1512 mm, so
$18x + 14y = 1512$
$$y = \frac{1512 - 18x}{14}$$
Total area A mm^2 is given by:
$A = 2y \times 6x$
Substituting $y = \dfrac{1512 - 18x}{14}$:
$$A = 12x\left(\frac{1512 - 18x}{14}\right)$$
$$= 1296x - \frac{108}{7}x^2$$

b For maximum area $\dfrac{\mathrm{d}A}{\mathrm{d}x} = 0$:

$$\frac{\mathrm{d}A}{\mathrm{d}x} = 1296 - \frac{216}{7}x$$
$$1296 - \frac{216}{7}x = 0$$
$$x = \frac{7 \times 1296}{216} = 42$$
When $x = 42$,
$$A = 1296 \times 42 - \frac{108}{7} \times 42^2$$
$$= 27\,216$$
Maximum area = 27 216 mm^2
(Check: $\dfrac{\mathrm{d}^2A}{\mathrm{d}x^2} = -\dfrac{216}{7} < 0 \quad \therefore$ maximum)